Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number

Candidate Number

Tuesday 25 June 2019

Morning (Time: 1 hour 30 minutes) | Paper Reference **9FM0/4C**

Further Mathematics
Advanced
Paper 4C: Further Mechanics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

Answer ALL questions. Write your answers in the spaces provided.

1.

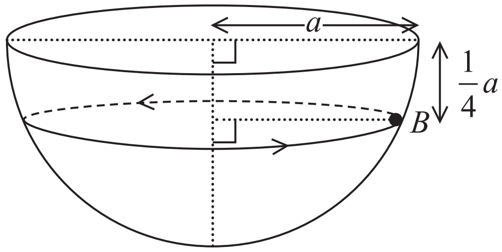
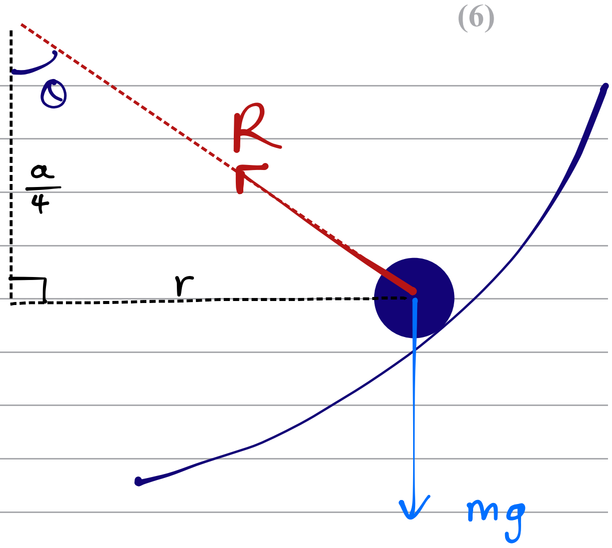


Figure 1

A hemispherical shell of radius a is fixed with its rim uppermost and horizontal. A small bead, B, is moving with constant angular speed, ω, in a horizontal circle on the smooth inner surface of the shell. The centre of the path of B is at a distance $\frac{1}{4}a$ vertically below the level of the rim of the hemisphere, as shown in Figure 1.

Find the magnitude of ω, giving your answer in terms of a and g.

(6)



Resolving Forces Vertically (↑),

$R\cos\theta = mg \longrightarrow (1)$

Resolving Forces Horizontally (←)

Net Force = centripetal force contributed by R

$R\sin\theta = mr\omega^2 \longrightarrow (2)$

$(2) \div (1)$: $\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{mr\omega^2}{mg} = \frac{r\omega^2}{g}$

Looking at the right angled triangle:

$\tan\theta = \frac{\text{opp.}}{\text{adj.}} = \frac{r}{a/4} = \frac{4r}{a}$

Equating the 2 expressions for $\tan\theta$:

$\frac{r\omega^2}{g} = \frac{4r}{a} \Rightarrow \omega^2 = \frac{4g}{a}$

$\therefore \omega = 2\sqrt{g/a}$

P 6 1 1 8 5 A 0 2 2 8

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 1 continued

(Total for Question 1 is 6 marks)

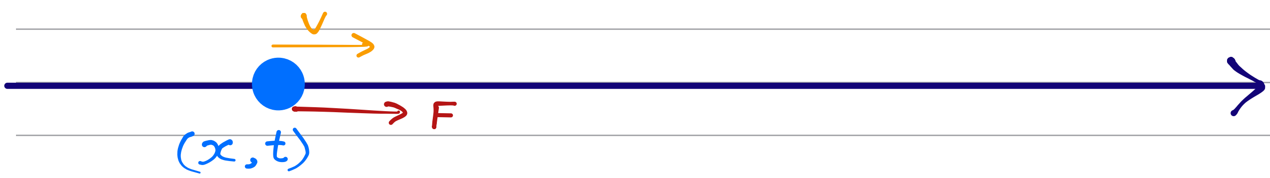
2. A particle, P, of mass 0.4 kg is moving along the positive x-axis, in the positive x direction under the action of a single force. At time t seconds, $t > 0$, P is x metres from the origin O and the speed of P is v m s^{-1}. The force is acting in the direction of x increasing and has magnitude $\frac{k}{v}$ newtons, where k is a constant.

At $x = 3$, $v = 2$ and at $x = 6$, $v = 2.5$

(a) Show that $v^3 = \frac{61x + 9}{24}$ **(6)**

The time taken for the speed of P to increase from 2 m s^{-1} to 2.5 m s^{-1} is T seconds.

(b) Use algebraic integration to show that $T = \frac{81}{61}$ **(4)**



a) Using Newton's 2nd Law on the particle,

$$F = ma$$

$$\frac{k}{v} = m\frac{dv}{dt}$$

$$\frac{k}{v} = m\frac{dv}{dx}\frac{dx}{dt} \qquad \text{[Using the Chain Rule]}$$

$$\frac{k}{v} = 0.4\frac{dv}{dx}v$$

$$0.4v^2\frac{dv}{dx} = k \qquad \text{[Separable ODE]}$$

$$\int 0.4v^2\,dv = \int k\,dx$$

$$\frac{0.4}{3}v^3 = kx + c$$

Using our 2 boundary conditions, we can solve for c and k

When $x = 3$, $v = 2$:

$$\frac{0.4}{3} \times 2^3 = 3k + c \Rightarrow 3k + c = \frac{16}{15} \longrightarrow (1)$$

When $x = 6$, $v = 2.5$:

$$\frac{0.4}{3} \times 2.5^3 = 6k + c \Rightarrow 6k + c = \frac{25}{12} \longrightarrow (2)$$

Question 2 continued

② − ① : $3k = \frac{61}{60}$

$\Rightarrow k = \frac{61}{180}$

From ①:

$\Rightarrow c = \frac{16}{15} - \frac{61}{60} = \frac{3}{60} = \frac{1}{20}$

Plug c and k into solution for v,

$v^3 = \frac{3}{0.4}\left(\frac{61x}{180} + \frac{1}{20}\right)$

$\therefore v^3 = \frac{61x + 9}{24}$

b) From Newton's 2nd Law:

$F = ma$

$\frac{k}{v} = m\frac{dv}{dt}$

$\frac{61}{180v} = \frac{2}{5}\frac{dv}{dt}$

$\Rightarrow \frac{dv}{dt} = \frac{61}{72v}$

$\int 72v\,dv = \int 61\,dt$

Let $t = 0$ when $v = 2$, so $t = T$ when $v = 2.5$

$\int_{2}^{2.5} 72v\,dv = \int_{0}^{T} 61\,dt$

Question 2 continued

$$\left[36v^2\right]_2^{2.5} = \left[61t\right]_0^T$$

$$36\left(2.5^2 - 2^2\right) = 61T$$

$$36 \cdot 2.25 = 61T$$

$$81 = 61T$$

$$T = \frac{81}{61}$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 2 continued

(Total for Question 2 is 10 marks)

3. Numerical (calculator) integration is not acceptable in this question.

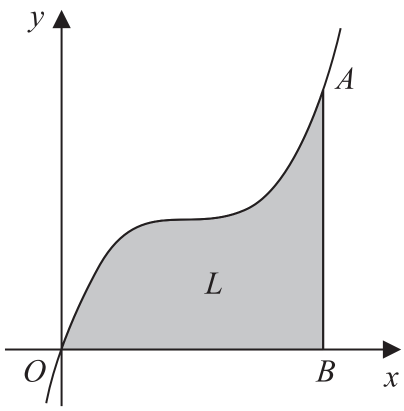


Figure 2

The shaded region OAB in Figure 2 is bounded by the x-axis, the line with equation $x = 4$ and the curve with equation $y = \frac{1}{4}(x-2)^3 + 2$. The point A has coordinates (4, 4) and the point B has coordinates (4, 0).

A uniform lamina L has the shape of OAB. The unit of length on both axes is one centimetre. The centre of mass of L is at the point with coordinates $(\bar{x}, \bar{y})$.

Given that the area of L is 8 cm^2,

(a) show that $\bar{y} = \frac{8}{7}$
(4)

The lamina is freely suspended from A and hangs in equilibrium with AB at an angle $\theta°$ to the downward vertical.

(b) Find the value of θ.
(7)

a) Since we are given areas and the lamina has a uniform mass density, we can use the formula for y-coord. of the centre of mass:

$$\bar{y} = \frac{\int_0^4 \frac{1}{2}y^2\,dx}{\int_0^4 y\,dx} = \frac{\frac{1}{2}\int_0^4 \left(\frac{1}{4}(x-2)^3+2\right)^2 dx}{8}$$

As we are told that the area of the lamina is 8

$$\Rightarrow \bar{y} = \frac{1}{16}\int_0^4 \frac{(x-2)^6}{16} + (x-2)^3 + 4 \quad dx$$

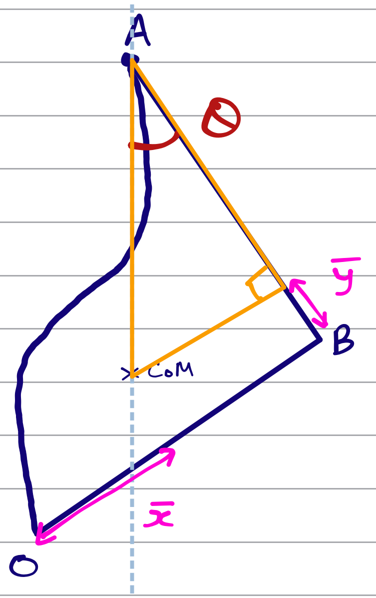
Question 3 continued

$$= \frac{1}{16}\left[\frac{(x-2)^7}{112} + \frac{(x-2)^4}{4} + 4x\right]_0^4$$

$$= \frac{1}{16}\left[\frac{128}{112} + \frac{16}{4} + 16 + \frac{128}{112} - \frac{16}{4}\right]$$

$$= \frac{1}{16} \times \frac{128}{7} = \frac{8}{7}$$

b)



Using a similar approach to a),

$$\bar{x} = \frac{\int_0^4 xy\,dx}{\int_0^4 y\,dx} = \frac{\int_0^4 \frac{x(x-2)^3}{4} + 2x\,dx}{8}$$

$$= \frac{1}{8}\left[\frac{x(x-2)^4}{16} - \frac{(x-2)^5}{80} + x^2\right]_0^4$$

$$= \frac{1}{8}\left[\frac{4\times 16}{16} - \frac{32}{80} + 16 - \frac{32}{80}\right]$$

$$= \frac{1}{8} \times \frac{96}{5} = \frac{12}{5}$$

By considering the right △,

$$\tan\theta = \frac{\text{opp.}}{\text{adj.}} = \frac{OB - \bar{x}}{AB - \bar{y}} = \frac{4 - 12/5}{4 - 8/7} = \frac{8/5}{20/7} = \frac{56}{100} = \frac{14}{25}$$

$$\Rightarrow \theta = \arctan\left(\frac{14}{25}\right) = 29.248826\ldots^\circ$$

$$\approx 29.2^\circ \text{ (To 2 sf)}$$

P61185A0928

Question 3 continued

Question 3 continued

(Total for Question 3 is 11 marks)

4. A flagpole, AB, is 4 m long. The flagpole is modelled as a non-uniform rod so that, at a distance x metres from A, the mass per unit length of the flagpole, $m\,\text{kg}\,\text{m}^{-1}$, is given by $m = 18 - 3x$.

(a) Show that the mass of the flagpole is 48 kg.

(3)

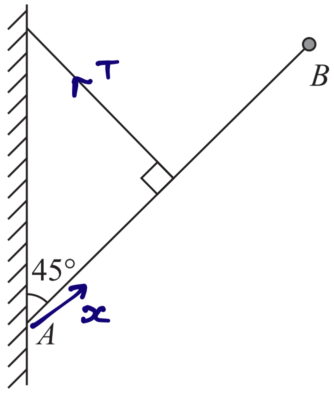


Figure 3

The end A of the flagpole is fixed to a point on a vertical wall. A cable has one end attached to the midpoint of the flagpole and the other end attached to a point on the wall that is vertically above A. The cable is perpendicular to the flagpole. The flagpole and the cable lie in the same vertical plane that is perpendicular to the wall. A small ball of mass 4 kg is attached to the flagpole at B. The cable holds the flagpole and ball in equilibrium, with the flagpole at 45° to the wall, as shown in Figure 3.

The tension in the cable is T newtons.

The cable is modelled as a light inextensible string and the ball is modelled as a particle.

(b) Using the model, find the value of T.

(8)

(c) Give a reason why the answer to part (b) is not likely to be the true value of T.

(1)

a) Total Mass $= \int$ Linear Mass Density dx

$$= \int_0^4 18 - 3x \; dx$$

$$= \left[18x - \frac{3}{2}x^2\right]_0^4$$

$$= 18\times4 - \frac{3}{2}\times4^2 - 0 + 0$$

$$= 72 - 24 = \underline{\underline{48}}\text{ kg}$$

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

P 6 1 1 8 5 A 0 1 2 2 8

DO NOT WRITE IN THIS AREA

Question 4 continued

b) To find the centre of mass $(x = d)$ of the rod,

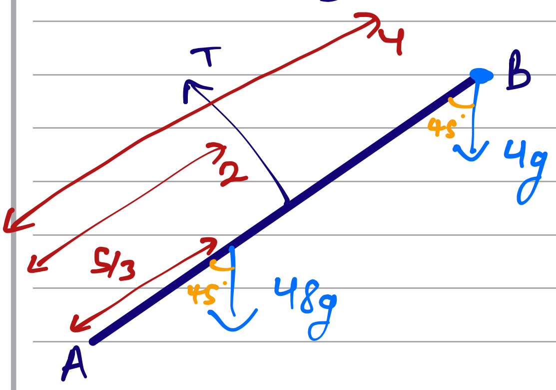
$$\underline{\int_0^4 x(18-3x)\,dx} = \int_0^4 18x - 3x^2\,dx = \left[9x^2 - x^3\right]_0^4$$

↳ integral of position weighted by mass density

$$= 9\times16 - 64$$
$$= 80$$

$\Rightarrow 48d = 80$

$d = \frac{5}{3}$ m



Now taking moments about A,

$$\sum \circlearrowleft = \sum \circlearrowright$$

$$2T = 4\cos(45^\circ)\times 4g + \frac{5}{3}\cos(45^\circ)\times 48g$$

$$2T = 8\sqrt{2}g + 40\sqrt{2}g$$

$$= 48\sqrt{2}g$$

$$T = 24\sqrt{2}g \approx \underline{\underline{333\ N}}\ \text{(To 3sf)}$$

c) The ball is modelled as a particle with point mass. In reality, the balls Centre of Mass may be further away from A

Question 4 continued

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 12 marks)

5.

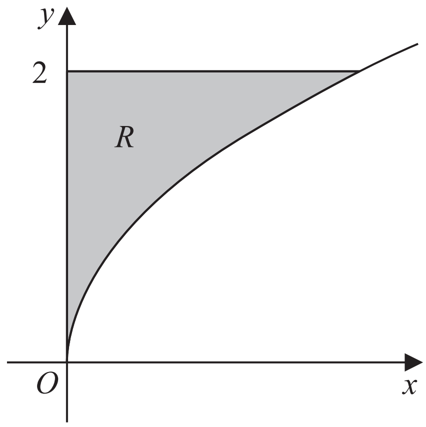


Figure 4

The region R, shown shaded in Figure 4, is bounded by part of the curve with equation $y^2 = 2x$, the line with equation $y = 2$ and the y-axis. The unit of length on both axes is one centimetre. A uniform solid, S, is formed by rotating R through 360° about the y-axis.

Given that the volume of S is $\frac{8}{5}\pi\,\text{cm}^3$,

(a) show that the centre of mass of S is $\frac{1}{3}$ cm from its plane face.

(4)

A uniform solid cylinder, C, has base radius 2 cm and height 4 cm. The cylinder C is attached to S so that the plane face of S coincides with a plane face of C, to form the paperweight P, shown in Figure 5. The density of the material used to make S is three times the density of the material used to make C.

$\rho_s = 3\rho_c$

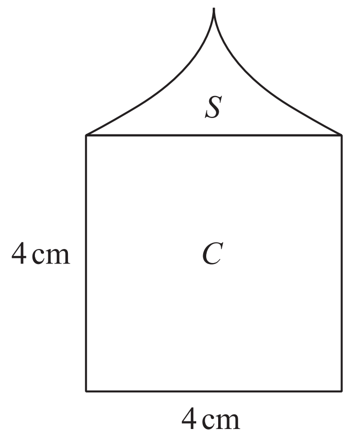


Figure 5

The plane face of P rests in equilibrium on a desk lid that is inclined at an angle $\theta°$ to the horizontal. The lid is sufficiently rough to prevent P from slipping. Given that P is on the point of toppling,

(b) find the value of θ.

(7)

DO NOT WRITE IN THIS AREA

Question 5 continued

a) Volume of Revolution $= \pi \int_0^2 x^2 \, dy = \frac{8}{5}\pi$ [given]

For a uniform solid of revolution,

$$\bar{y} = \frac{\pi \int_0^2 x^2 y \, dy}{\pi \int_0^2 x^2 \, dy} \leftarrow \text{We're given the denominator}$$

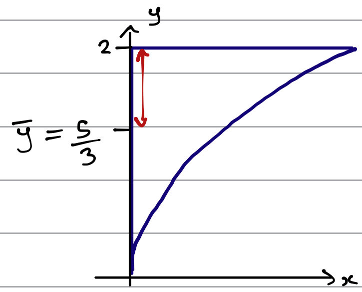
To evaluate the numerator:

$$\pi \int x^2 y \, dy = \pi \int_0^2 \left(\frac{y^2}{2}\right)^2 y \, dy = \frac{\pi}{4} \int_0^2 y^5 \, dy$$

$$= \frac{\pi}{4}\left[\frac{y^6}{6}\right]_0^2 = \frac{64\pi}{24}$$

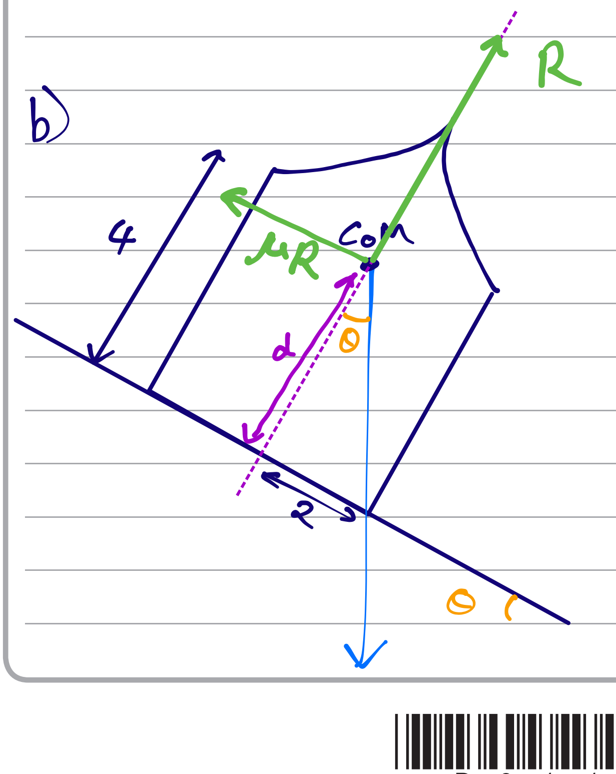
$$= \frac{8\pi}{3}$$

$$\Rightarrow \bar{y} = \frac{\frac{8}{3}\pi}{\frac{8}{5}\pi} = \frac{5}{3} \text{ cm}$$



Distance to plane face $= 2 - \frac{5}{3} = \frac{1}{3}$ cm

b)



At toppling angle,

Since the axes of symmetry of both pieces coincide, the centre of mass will lie along the axis of symmetry of the final shape

Question 5 continued

Using mass ratios :

Shape	Mass Ratio	Distance from Centre of Mass to base
S	$3 \times \frac{8\pi}{5} = \frac{24\pi}{5}$	$4 + \frac{1}{3} = \frac{13}{3}$
C	$\pi(2)^2 \times 4 = 16\pi$	2
P	$\left(16 + \frac{24}{5}\right)\pi = \frac{104}{5}\pi$	d

Taking Moments about the diameter of the base :

$$\sum \circlearrowright = \frac{24\pi}{5} \times \frac{13}{3} + 16\pi \times 2 \equiv \frac{104\pi}{5} d$$

$$\Rightarrow d = \frac{264}{104} = \frac{33}{13} \text{ cm}$$

From the right Δ :

$$\tan\theta = \frac{\text{opp.}}{\text{adj.}} = \frac{2}{d} = \frac{26}{33}$$

$$\theta = \arctan\left(\frac{26}{33}\right) \approx 38.2^\circ \text{ (to 3sf)}$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

(Total for Question 5 is 11 marks)

6. The points A and B lie on a smooth horizontal surface with $AB = 4.5\,\text{m}$.

A light elastic string has natural length 1.5 m and modulus of elasticity 15 N. One end of the string is attached to A and the other end of the string is attached to B. A particle, P, of mass 0.2 kg, is attached to the stretched string so that APB is a straight line and $AP = 1.5\,\text{m}$. The particle rests in equilibrium on the surface.

The particle is now moved directly towards A and is held on the surface so APB is a straight line with $AP = 1\,\text{m}$.

The particle is released from rest.

(a) Prove that P moves with simple harmonic motion.

(5)

(b) Find

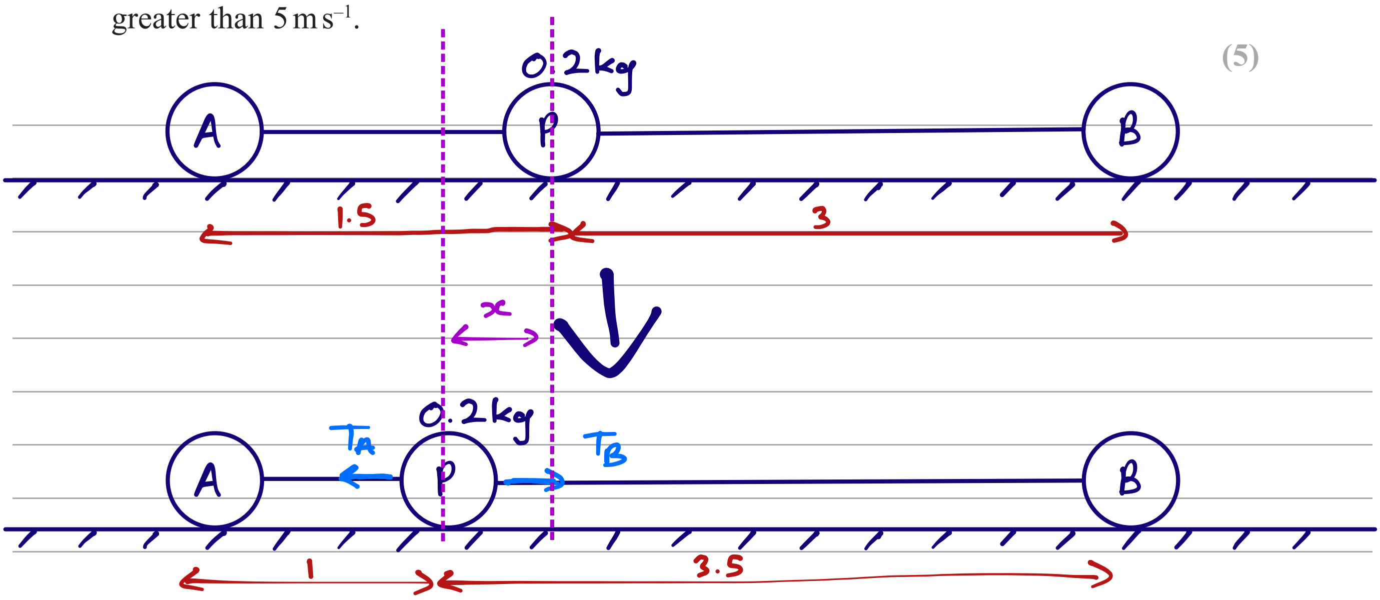
(i) the maximum speed of P during the motion,

(ii) the maximum acceleration of P during the motion.

(3)

(c) Find the total time, in each complete oscillation of P, for which the speed of P is greater than $5\,\text{m}\,\text{s}^{-1}$.

(5)



a) Using Newton's 2^{nd} Law on P :

$$\sum F = ma$$

$$T_B - T_A = -0.2 \times \frac{d^2x}{dt^2}$$

Using Hooke's Law, we know T_A and T_B :

$$\frac{15(2+x)}{1} - \frac{15(1-x)}{0.5} = -0.2 \frac{d^2x}{dt^2}$$

P 6 1 1 8 5 A 0 2 0 2 8

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 6 continued

$$45x + 30 - 30 = -0.2\frac{d^2x}{dt^2}$$

$$\Rightarrow \frac{d^2x}{dt^2} = -225x$$

This equation of motion is of the form $\ddot{x} = -\omega^2 x$ (as the acceleration is proportional and in the opposite direction to x)

b) Amplitude = Initial displacement of P = 0.5 m = A

$\omega = \sqrt{225} = 15 \ s^{-1}$

Max speed = $A\omega = 0.5 \times 15 = 7.5\ ms^{-1}$

Max acc. = $A\omega^2 = 0.5 \times 225 = 112.5\ ms^{-2}$

c) SHM solution: $x = A\cos(\omega t)$

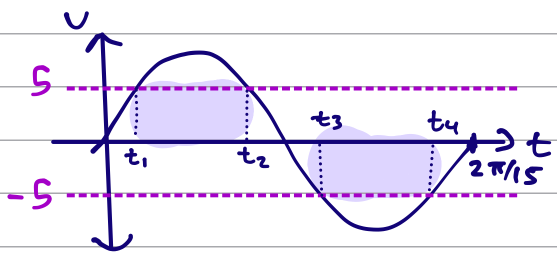
$\Rightarrow x = 0.5\cos(15t)$

$\Rightarrow \frac{dx}{dt} = -7.5\sin(15t)$

When $\left|\frac{dx}{dt}\right| = 5$,

$7.5\sin(15t) = 5$

$\Rightarrow \sin(15t) = \frac{2}{3}$



$t_1 = \frac{1}{15}\arcsin\left(\frac{2}{3}\right) = 0.0486485\ldots$ s

$t_2 = \frac{\pi}{15} - 0.04864\ldots = \frac{\pi}{15} - t_1$

Total time $|v| > 5 = (t_2 - t_1) + (t_4 - t_3)$

$= 2(t_2 - t_1)$ [By symmetry]

$= 2\left(\frac{\pi}{15} - 2t_1\right) = 0.2242849\ldots$ s

≈ 0.22 s

Question 6 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 6 continued

(Total for Question 6 is 13 marks)

7. A particle, P, of mass m is attached to one end of a light rod of length L. The other end of the rod is attached to a fixed point O so that the rod is free to rotate in a vertical plane about O. The particle is held with the rod horizontal and is then projected vertically downwards with speed u. The particle first comes to instantaneous rest at the point A.

(a) Explain why the acceleration of P at A is perpendicular to OA.
(1)

At the instant when P is at the point A the acceleration of P is in a direction making an angle θ with the horizontal. Given that $u^2 = \dfrac{2gL}{3}$,

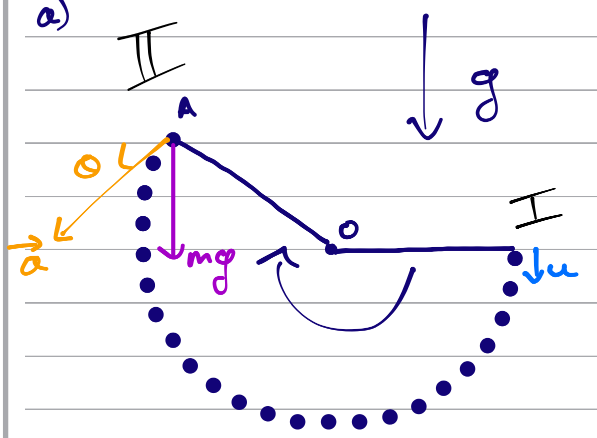
(b) find

(i) the magnitude of the acceleration of P at the point A,

(ii) the size of θ.
(6)

(c) Find, in terms of m and g, the magnitude of the tension in the rod at the instant when P is at its lowest point.
(5)

a)



When the particle reaches A, it has 0 speed.

$V = 0 \Rightarrow$ centripetal acc. $= \frac{0^2}{L} = 0$

$\therefore$ There is no acceleration towards O at this instant

$\Rightarrow$ Acceleration must be $\perp$ to OA (zero component along OA)

b) Using conservation of energy

KE at I = GPE difference between II and I

$\frac{1}{2}mu^2 = mgL\cos\theta$

$\frac{1}{2} \times \frac{2gL}{3} = gL\cos\theta$

$\theta = \arccos\left(\frac{1}{3}\right) = 70.52877...^\circ \approx 71^\circ$ (To 2sf)

P 6 1 1 8 5 A 0 2 4 2 8

DO NOT WRITE IN THIS AREA

Question 7 continued

Magnitude of acc. = Component of Weight $\perp$ OA

$= g \sin\theta$

$\cos\theta = \frac{1}{3} \Rightarrow \sin\theta = \sqrt{1-\cos^2\theta}$

$= \sqrt{8/9}$

$= \frac{2\sqrt{2}}{3}$

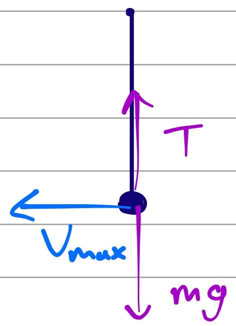
$\therefore |\vec{a}| = \frac{2\sqrt{2}}{3} g$

c) Since the mass is undergoing circular motion at its lowest point (i.e. acceleration $\parallel$ OA),

$\Rightarrow \sum F$ = centripetal force

$T - mg = \frac{mv_{max}^2}{L}$

$\Rightarrow T = mg + \frac{mv_{max}^2}{L}$



To find V_{max},

KE gained = GPE Lost

$\frac{1}{2}mv_{max}^2 - \frac{1}{2}mu^2 = mgL$

$v_{max}^2 = 2gL + u^2 = 2gL + \frac{2gL}{3} = \frac{8gL}{3}$

$\Rightarrow T = mg + \frac{m}{L} \times \frac{8gL}{3} = \frac{11}{3}mg$

Question 7 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

Question 7 continued

(Total for Question 7 is 12 marks)

TOTAL FOR PAPER IS 75 MARKS